Chapter One

Technical Underpinnings in Mathematics and Physics

The field of building science is based on scientific concepts and principles. This does not mean, however, that an in-depth knowledge of science and mathematics is necessarily required for the application of sound building science principles during the building design process. In most cases an understanding of the higher-level technical notions involved is sufficient for the designer to make the necessary decisions during the early design stages, when the conceptual design solution is formulated. However, it is most important that those decisions are sound so that they can be translated into detailed solutions during later design stages by consultants with specialized expertise.

Accordingly, the purpose of this book is to describe and explain the underlying concepts and principles of the thermal, lighting, and acoustic determinants of building design, without delving into the detailed methods that are applied by engineers and other technical consultants to design and implement detailed system solutions. Nevertheless, there are some basic mathematical methods and scientific principles that the reader should be familiar with to easily follow the largely qualitative treatment of the subject matter of the subsequent chapters. The particular mathematical methods that are briefly reviewed include elementary linear equations and *normal distribution* statistics. In respect to Physics the fundamental concepts related to temperature scales and *black body* radiation have been selected for explanation in this introductory chapter, because they form the basis of discussions related to the thermal determinants of building design and artificial light sources, respectively.

1.1 Linear Equations

Many problem systems in environmental design, planning, engineering, and management may be defined in terms of a set of equations or algorithms that can be solved mathematically. Naturally, the method of solution, the kind of solutions, and the question of solvability (i.e., whether or not the set of equations has a solution) will depend largely on the types of equations involved. We shall therefore briefly describe a simple, but also quite typical, type of equation known as the *linear equation*.

Also referred to as *first degree* equations, linear equations obey the following two rules:

Rule 1: All variables or unknown quantities are to the exponent 1. Therefore, the equation $x^2 - 4y = 1$ is not a linear equation (because the variable x is raised to power 2).

Rule 2: Variable or unknown quantities appear only once in each term. For example, in the equation $ax + by + cz = k$ each of the terms on the left side of the equation contains a constant (i.e., a, b and c) and a variable (i.e., x, y and z), but there is never more than one variable in any of the terms. Therefore, this is a linear equation with three variables. However, in the equation $8xy = -14$ the variables x and y appear in the same term and therefore this is not a linear equation. If we were to plot this equation on graph paper with y on the vertical axis and x on the horizontal axis for multiple values of x and y then the resulting graph would be a curve. On the other hand a linear equation, by virtue of its name, will always result in a straight line when plotted on graph paper.

The general form of a linear equation is $A_1X_1 + A_2X_2 + A_3X_3 + \ldots A_iX_i = C$ where A_1 to A_i and C are constants and X_1 to X_i are variables. The following are examples of linear equations and all of these will result in straight lines when plotted on graph paper:

$$7x = -16; \text{ or } 2x - 6y = 8; \text{ or } x - 3y + 17z = -3$$

There is another very useful rule in Algebra that applies to not just linear equations, but all equations.

Rule 3: Whatever operation is applied to one side of an equation must also be applied to the other side. Restating this rule in a more positive form: any mathematical operation such as multiplication, division, addition, and subtraction can be applied to an equation as long as it is applied to both sides of the equation. We will use this rule repeatedly in Section 1.1.2 to solve equations involving two unknowns.

1.1.1 What are Unknown Quantities?

It is generally considered convenient in Algebra to categorize equations according to the number of unknown quantities (or more commonly the number of *unknowns*). This refers simply to the number of different variables contained in the equation. For example:

$12x - 16 = 0$ .... has *1* unknown

$2x + 17y = -66$ .... has *2* unknowns

$-114x + 212y = 22z + 9$ .... has *3* unknowns

$A_1X_1 + A_2X_2 + A_3X_3 + \ldots A_iX_i = C$ .... has *i* unknowns

A set or system of equations that are to be considered together for the solution of the same problem are known as *simultaneous equations*. It is a fundamental and very important rule in mathematics that to be able to solve a system of simultaneous equations there *must be at least as many equations as there are unknowns*.

1.1.2 Simultaneous Equations with Two Unknowns

A problem that has only two linear variables can be solved quite easily in Algebra as a set of two simultaneous equations. The approach is to eliminate one of the two unknowns by utilizing one of the following three alternative methods:

Method A: Elimination by addition or subtraction. Multiply one or both of the equations by a constant and then add or subtract the equations to eliminate one of the unknowns. For example, solve the following two equations for the unknowns x and y: $5x + 2y = 32$ and $2x - y = 2$.

$5x + 2y = 32$ (1)

$4x - 2y = 4$ (2) multiply equation (2) by *2*

$9x = 36$ add equation (2) to equation (1)

$\mathbf{x = 4}$	divide both sides of the equation by *9*
$20 + 2y = 32$	substitute for $x = 4$ in equation (1)
$2y = 12$	subtract *20* from both sides of the equation
$\mathbf{y = 6}$	divide both sides of the equation by *2*

Method B: Elimination by substitution. Using one of the equations, find the value of one unknown in terms of the other, then substitute. For example, solve the following two equations: $2x + 4y = 50$ and $3x + 5y = 66$:

$2x + 4y = 50$ (1)	
$3x + 5y = 66$ (2)	
$2x = 50 - 4y$	subtract *4y* from both sides of equation (1)
$x = 25 - 2y$	divide both sides by *2* to find *x* in terms of *y*
$3(25 - 2y) + 5y = 66$	substitute for *x* in equation (2)
$75 - 6y + 5y = 66$	expand the brackets on the left side
$-y = -9$	subtract *75* from both sides of the equation
$\mathbf{y = 9}$	multiply both sides of the equation by *-1*
$2x + (4 \times 9) = 50$	substitute for $y = 9$ in equation (1)
$2x + 36 = 50$	expand the brackets on the left side
$2x = 14$	subtract *36* from both sides of the equation
$\mathbf{x = 7}$	divide both sides of the equation by *2*

Method C: Elimination by comparison. From each equation find the value of one of the unknowns in terms of the other and then form an equation of these equal values. For example, solve: $3x + 2y = 27$ and $2x - 3y = 5$:

$3x + 2y = 27$ (1)	
$2x - 3y = 5$ (2)	
$3x = 27 - 2y$	subtract *2y* from both sides of equation (1)
$x = (27 - 2y)/3$	divide both sides of the equation by *3*
$2x = 5 + 3y$	add *3y* to both sides of equation (2)
$x = (5 + 3y)/2$	divide both sides of the equation by *2*
$(27 - 2y)/3 = (5 + 3y)/2$	equate the two values of *x*
$27 - 2y = 3(5 + 3y)/2$	multiply both sides of the equation by *3*
$54 - 4y = 15 + 9y$	multiply both sides of the equation by *2*
$-4y = -39 + 9y$	subtract *54* from both sides of the equation
$-13y = -39$	subtract *9y* from both sides of the equation
$13y = 39$	multiply both sides of the equation by *-1*
$\mathbf{y = 3}$	divide both sides of the equation by *13*
$3x + 6 = 27$	substitute for $y = 3$ in equation (1)
$3x = 21$	subtract *6* from both sides of the equation
$\mathbf{x = 7}$	divide both sides of the equation by *3*

In all of these examples we have dealt with simultaneous equations that have only two unknowns and already the solution methods described become rather tedious. In architectural design, building science and construction management many of the problems encountered such as the structural analysis of a building frame, or the analysis of an electrical circuit, or the solution of a work flow management problem will often involve a set of linear equations with three or more unknowns. Such systems of equations are normally solved using methods that require the equations to be formulated as a matrix of variables and constants, as shown below.

$$\begin{array}{l}
A_{11}X_1 + A_{12}X_2 + \ldots\ldots\ldots \; A_{1n}X_n = C_1 \\
A_{21}X_1 + A_{22}X_2 + \ldots\ldots\ldots \; A_{2n}X_n = C_2 \\
A_{31}X_1 + A_{32}X_2 + \ldots\ldots\ldots \; A_{3n}X_n = C_3 \\
\quad . \quad . \quad . \quad . \quad + \qquad\qquad . \quad . \quad . \\
\quad . \quad . \quad . \quad . \quad + \qquad\qquad . \quad . \quad . \\
A_{n1}X_1 + A_{n2}X_2 + \ldots\ldots\ldots \; A_{nn}X_n = C_n
\end{array}$$

Where: A_{11} to A_{nn} are the known coefficients of the unknowns (or variables)

C_1 to C_n are the known constants

X_1 to X_n are the unknowns for which the equations are to be solved

The subscripted format is also referred to as an array and is a very convenient mathematical notation for representing problems that involve many linear relationships.

1.2 Some Statistical Methods

The word *statistics* was first applied to matters of government dealing with the quantitative analysis of births, deaths, marriages, income, and so on, necessary for effective government planning. Today, statistics is applied in a number of ways to any kind of objective or subjective data, whether this be a small sample or the total available information. There are basically two kinds of statistics:

Descriptive statistics deal with the classification of data, the construction of histograms and other types of graphs, the calculation of means, and the analysis of the degree of scatter within a given sample.

Inferential statistics may be described as the science of making decisions when there is some degree of uncertainty present (in other words, making the best decision on the basis of incomplete information).

For example, a large contracting firm may wish to embark on the manufacture of standard, mass-produced, precast concrete balustrades for staircases, balconies, and similar structures. On the assumption that the required height of a comfortable balustrade is directly related to the heights of human beings, the contracting firm considers it necessary to conduct a survey of the heights of potential users in various countries of the world. Obviously, to measure the height of every potential user (even if this were possible) would be very costly and time consuming. Instead, a small number of potential users constituting a sample of the total population are selected for measurement. The selection is usually by a random process, although a number of other kinds of sampling procedures exist. However, what is most useful and important is that on the basis of this relatively small set of measurements we are able to make predictions about the range and distribution of heights of persons in the sampled countries. The accuracy of our predictions will depend more on the representativeness than the size of the sample (Figure 1.1).

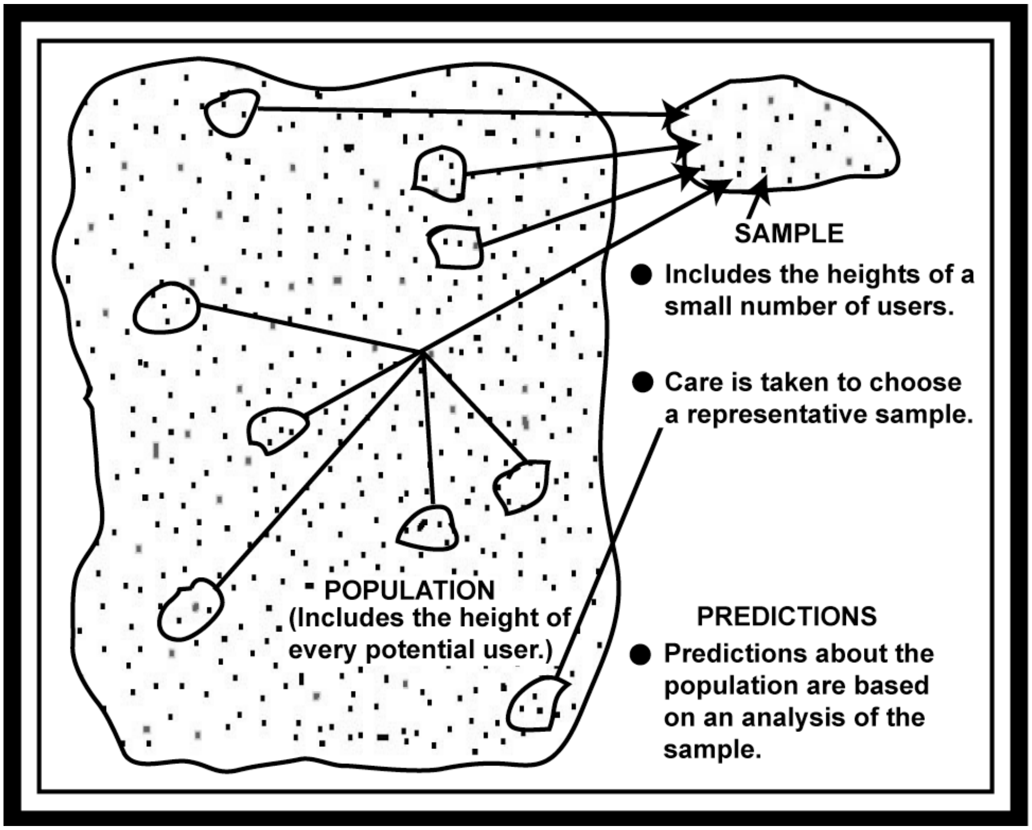

Figure 1.1: Statistical sampling

1.2.1 Ordering Data

It is very difficult to learn anything by examining unordered and unclassified data. Let us assume that in the example under consideration the following sample (Table 1.1) of the heights of persons has been collected (i.e., measured to the nearest inch):

Table 1.1: Sample of the heights of persons (in inches)

72	67	65	70	82	76	60	62	68	59
50	78	67	68	68	68	64	80	54	49
67	64	71	75	60	70	69	69	65	79
67	69	65	69	68	78	59	64	72	72
81	76	52	53	56	82	71	68	63	59

These measurements may be represented graphically in the form of a histogram (i.e., bar chart) or distribution curve as shown in Figures 1.2 and 1.3, respectively. To facilitate the preparation of either of these two graphs it is convenient to prepare a frequency distribution table, in which the measurements are grouped into clearly defined classes. Class limits are carefully chosen so that no measurement can be allocated to more than one class (Table 1.2).

Table 1.2: Frequency distribution table

Class No.	Class Limits	Class Frequency
1	45.5 to 50.5	2
2	50.5 to 55.5	3
3	55.5 to 60.5	6
4	60.5 to 65.5	8
5	65.5 to 70.5	16
6	70.5 to 75.5	6
7	75.5 to 80.5	6
8	80.5 to 85.5	3

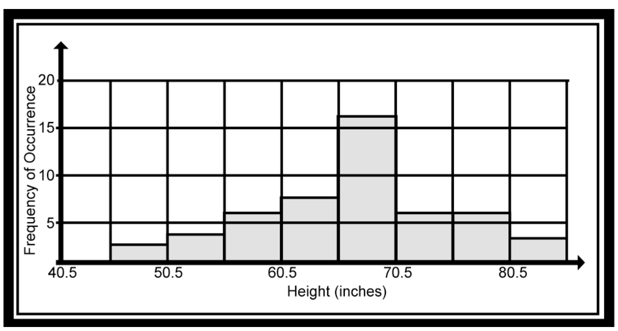

Figure 1.2: Histogram (or Bar Chart)

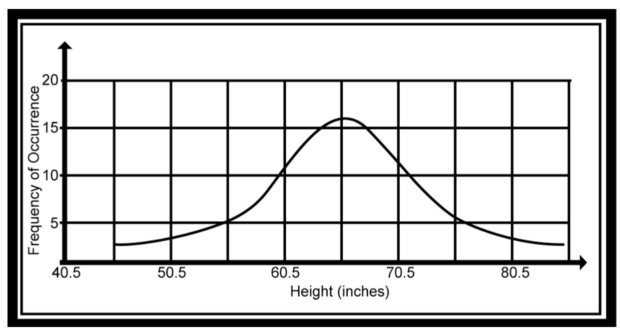

Figure 1.3: Distribution Curve

In addition to the construction of graphical representations of data there are a number of arithmetically calculated measures of central tendency that are frequently used to convey a quantitative sense of a set of data with a single numerical value. Four of these are discussed below in order of increasing importance:

Mid-Range is the value halfway between the smallest and largest observation. For the sample of human heights in Table 1.1 the mid-range is calculated to be:

Mid-Range = (49 + 82) / 2 = **65.5 IN**

Mode is defined as the observation in the sample that occurs most frequently. This means, of course, that some sets of data may not have a mode because no single value occurs more than once. In the sample shown in Table 1.1 there are several heights that occur more than once.

Mode = **68 IN** (occurs six times)

Median is defined as the middle observation if the sample observations are arranged in order from smallest to largest. Again, with reference to the sample shown in Table 1.1:

49, 50, 52, 53, 54, 56, 59, 59, 59, 60, 60, 62, 63, 64, 64, 64, 65, 65, 65, 67, 67, 67, 67, 68, **68**, **68**, 68, 68, 68, 69, 69, 69, 69, 70, 70, 71, 71, 72, 72, 72, 75, 76, 76, 78, 78, 79, 80, 81, 82, 82

But the total number of observations in Table 1.1 is 50, which is an even number. Therefore there are two middle observations. Typically, under these conditions the median is calculated to be halfway between the two middle observations. In this case the two middle observations are the same.

Median = (68 + 68) / 2 = **68 IN**

Mean (or Arithmetic Mean) is the average value of the sample. It is calculated simply by adding the values of all observations and dividing by the number of observations. In reference to Table 1.1:

Mean ($\bar{x}$) = 3360 / 50 = **67.2 IN**

The manual calculation of a mean using this method tends to become tedious if the sample is quite large, as it is in this case. Therefore, for samples that contain more than 30 observations, it is common practice to draw up a frequency distribution table with an expanded set of columns as shown in Table 1.3 below.

Table 1.3: Expanded frequency distribution table

Group Boundaries	Group Mid-value	f	t	(ft)
45.5 to 50.5	48	2	-3	-6
50.5 to 55.5	53	3	-2	-6
55.5 to 60.5	58	6	-1	-6
60.5 to 65.5	63	8	0	0
65.5 to 70.5	68	16	+1	+16
70.5 to 75.5	73	6	+2	+12
75.5 to 80.5	78	6	+3	+18
80.5 to 85.5	83	3	+4	+12
		∑ f = 50		**∑ (ft) = 40**

Where: t = [group mid-value - assumed Mean ($\bar{x}_o$)] / class interval (c)
$\bar{x}_o$ = any assumed Mean (63 in this case)
c = the class interval (5 in this case)

Based on the frequency distribution table the true mean ($\bar{x}$) of the sample is given as a function of an assumed mean ($\bar{x}_o$) plus a positive or negative correction factor.

$$\textbf{Mean } (\bar{x}) = \bar{x}_0 + c\,[\textstyle\sum (ft)] / [\sum ft] \quad \ldots\ldots\ldots\ldots \quad \textbf{(1.1)}$$

Applying equation 1.1 to the sample of heights shown in Tables 1.1 and 1.3 we calculate the true mean ($\bar{x}$) of the sample to be:

$$\bar{x} = 63 + 5\,(40 / 50)$$
$$\bar{x} = 63 + 4.0$$
$$\bar{x} = \underline{\textbf{67.0 IN}}$$

It should be noted that the smaller the class interval the more accurate the result will be. In this case, with a class interval (c) of 5 the error is 0.2 (or 0.3%).

1.2.2 The Normal Distribution Curve

We have seen that frequency distributions are of great value for the statistical analysis of data. Moreover, there would appear to be considerable merit in the standardization of frequency distributions leading, for example, to the tabulation of coordinates and so on. In fact, such systems have been devised. One of these relies on a rather distinctive natural phenomenon. There are a large number of distributions that appear to have a symmetrical, bell-shaped distribution (e.g., the heights, intelligence, and ages of persons) of the type shown in Figure 1.4.

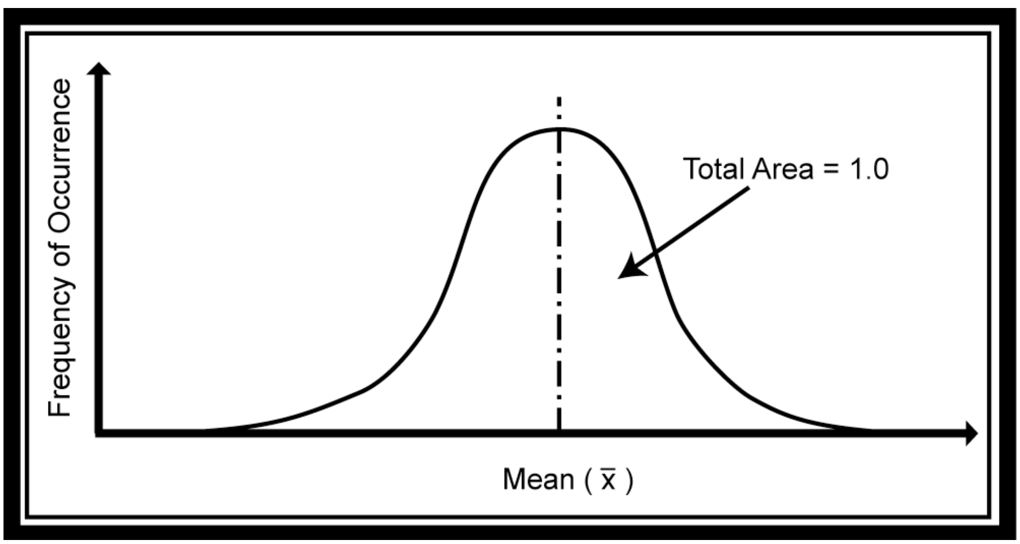

Figure 1.4: The Normal Distribution curve

This unique distribution, which is known as the Normal Distribution curve, or the Error Law, or the Gaussian curve, occupies a prominent position in statistical theory. It has the following characteristics:

- The total area under the normal distribution curve is assumed to be unity (i.e. 1.0), as shown in Figure 1.4.
- The exact shape of the curve may vary from distribution to distribution although the area will always remain the same (Figure 1.5).
- The Normal Distribution curve has been arbitrarily divided into three major sections (with subsections), so that judgments may be made regarding the

exact variation (i.e., the shape of the curve) for each distribution. These sections are defined as standard deviations from the mean (Figure 1.6).

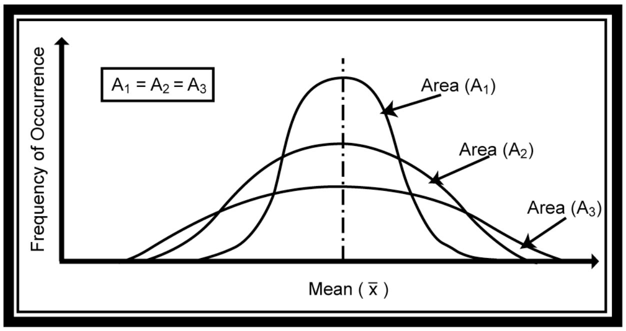

Figure 1.5: Various distributions

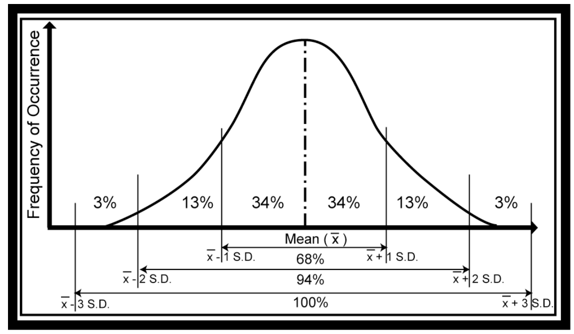

Figure 1.6: Standard Deviations

Accordingly, the Standard Deviation (SD) of a sample provides a method for calculating the amount of scatter or variation in a sample. For example, we are readily able to distinguish between the two Normal Distributions shown in Figure 1.7, by reference to their standard deviations.

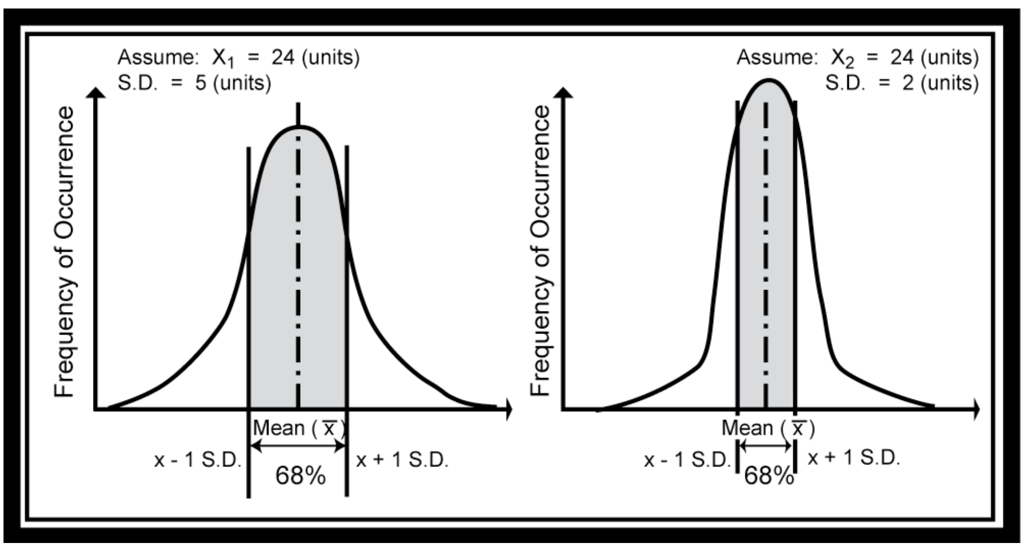

Figure 1.7: Two different Normal Distributions

For the first distribution 68% of the sample observations lie between 19 units (i.e., 24 - 5) and 29 units (i.e., 24 + 5). In the case of the second distribution 68% of the sample observations lie between 22 units (i.e., 24 - 2) and 26 units (i.e., 24 + 2). Obviously, the first distribution has greater variation among the observations of the sample than the second distribution. The calculation of the standard deviation of a sample is basic to virtually all statistical procedures dealing with the Normal Distribution curve. It allows us to proceed with further predictions relating to the degree of scatter or variation likely to be encountered in the population from which the sample was drawn, and the probable accuracy of these predictions.

1.2.3 The Standard Deviation of a Sample

There are basically three methods available for the calculation of the Standard Deviation of a sample. The first method is used whenever a Frequency Distribution table has been drawn up (i.e., when the number of observations in the sample is large). In reference to the sample of the heights of persons discussed previously in Section 1.2.1 (Table 1.1), the Frequency Distribution table may be extended to calculate the Standard Deviation of the sample of 50 measured heights according to the following formula:

$$\mathbf{S = c\,[\,(\,(\sum(ft^2)\,/\,\sum(f)\,)\,-\,(\,(\sum(ft)\,/\,\sum(f)\,)^2\,]^{1/2}} \quad \text{..................................} \quad \mathbf{(1.2)}$$

Group Boundaries	**Group Mid-value**	**f**	**t**	**(ft)**	**(ft^2)**
45.5 to 50.5	48	2	-3	-6	18
50.5 to 55.5	53	3	-2	-6	12
55.5 to 60.5	58	6	-1	-6	6
60.5 to 65.5	63	8	0	0	0
65.5 to 70.5	68	16	+1	+16	16
70.5 to 75.5	73	6	+2	+12	24
75.5 to 80.5	78	6	+3	+18	54
80.5 to 85.5	83	3	+4	+12	48
		$\sum$ f = 50		**$\sum$ (ft) = 40**	**$\sum$ (ft^2) = 178**

$$S = 5\,[\,(178\,/\,50\,) - (\,40\,/\,50\,)^2\,]^{1/2}$$
$$S = 5\,[\,3.56 - (\,0.8\,)^2\,]^{1/2}$$
$$S = 5\,[\,3.56 - 0.64\,]^{1/2}$$
$$S = 5\,[\,2.92\,]^{1/2}$$
$$S = 5\,[\,1.71\,]$$
$$\mathbf{S = 8.55\ IN}$$

Accordingly, with a Mean of 67.2 IN and a Standard Deviation of 8.55 IN we have now defined the sample within the following boundaries:

68% of the measured heights will lie in the range:
(67.2 - 8.55) to (67.2 + 8.55); i.e., **58.7 to 75.8 IN**

94% of the measured heights will lie in the range:
(67.2 - 17.1) to (67.2 + 17.1); i.e., **50.1 to 84.3 IN**

100% of the measured heights will lie in the range:
(67.2 - 25.7) to (67.2 + 25.7); i.e., **41.5 to 92.9 IN**

The second method for calculating the Standard Deviation of a sample is often used when the size of the sample is greater than 10 but less than 30 (i.e., a Frequency Distribution table has not been drawn up).

$$\mathbf{S = [\,(\sum(x^2) / \sum(f)\,) - (\bar{x})^2\,]^{1/2} \quad \ldots\ldots \quad (1.3)}$$

Where: x = each observation in sample

$\sum f$ = total number of observations

$\bar{x}$ = Mean of sample

Let us consider the following sample, containing measurements of the ultimate compressive strengths of 10 concrete test cylinders.

i.e., 2000, 2500, 4000, 1800, 2100, 3000, 2600, 2000, 2900, and 1900 psi

x	**x^2**
2000	4,000,000
2500	6,250,000
4000	16,000,000
1800	3,240,000
2100	4,410,000
3000	9,000,000
2600	6,760,000
2000	4,000,000
2900	8,410,000
1900	3,610,000
$\sum(x)$ = 24,800	**$\sum(x^2)$ = 65,680,000**

Step (1) - find the Mean ($\bar{x}$):

$\bar{x} = \sum(x) / \sum(f)$

$\bar{x}$ = 24,800 / 10

$\bar{x}$ = **2,480 psi**

Step (2) – find the Standard Deviation (S):

$S = [\,(65680000 / 10) – (2480)^2\,]^{1/2}$

$S = [\,(6.568 \times 10^6) – 6.150 \times 10^6\,]^{1/2}$

$S = [\,0.418 \times 10^6\,]^{1/2}$

$S = [\,41.8 \times 10^4\,]^{1/2}$

S = 646.5 psi

The third method is often used when the sample is very small (i.e., less than 10). For example, consider the following measurements taken of the permanent expansion of six brick panels (in thousandths of an inch):

i.e., 22, 24, 26, 28, 25, and 22 x 10^{-3} IN

$$\mathbf{S = [\,(\sum(x - \bar{x})^2\,) / \sum(f)\,]^{1/2} \quad \ldots\ldots \quad (1.4)}$$

Where: x = each observation in sample

$\sum(f)$ = total number of observations

$\bar{x}$ = Mean of sample (i.e., $\bar{x} = (147 / 6) \times 10^{-3} = 24.5 \times 10^{-3}$ IN)

$$S = [((22-24.5)^2 + (24-24.5)^2 + (26-24.5)^2 + (28-24.5)^2 + (25-24.5)^2 + (22-24.5)^2) / 6]^{1/2} \times 10^{-3}$$

$$S = [((-2.5)^2 + (-0.5)^2 + (1.5)^2 + (3.5)^2 + (0.5)^2 + (-2.5)^2) / 6]^{1/2} \times 10^{-3}$$

$$S = [(6.25 + 0.25 + 2.25 + 12.25 + 0.25 + 6.25) / 6]^{1/2} \times 10^{-3}$$

$$S = [27.5 / 6]^{1/2} \times 10^{-3}$$

$$S = [4.58]^{1/2} \times 10^{-3}$$

$$\mathbf{S = 2.14 \times 10^{-3}\ IN}$$

The square of the Standard Deviation is referred to as the Variance and is therefore another measure of the degree of scatter within a sample.

1.2.4 The Standard Deviation of the Population

Having calculated the Standard Deviation of a sample with any one of the three methods available (i.e., equations (1.2), (1.3) or (1.4) in Section 1.2) we are able to predict the Standard Deviation of the entire population (i.e., all possible observations) from which the sample has been drawn. If the Standard Deviation of the sample is *S*, then the best estimate of the Standard Deviation of the population (σ) is given by:

$$\sigma = S [\sum(f) / (\sum(f)-1)]^{1/2} \quad \ldots\ldots (1.5)$$

Obviously, the value of the correction factor $[\sum(f)/(\sum f-1)]^{1/2}$ is very much influenced by the size of the sample (i.e., $\sum(f)$). For example:

If the sample size is 6 …… then $[\sum(f) / (\sum(f) -1)]^{1/2} = 1.096$

If the sample size is 30 ….. then $[\sum(f) / (\sum(f) -1)]^{1/2} = 1.017$

If the sample size is 100 … then $[\sum(f) / (\sum(f) -1)]^{1/2} = 1.005$

If the sample size is 900 … then $[\sum(f) / (\sum(f) -1)]^{1/2} = 1.001$

Accordingly, samples containing 30 or more observations are normally considered to be large samples, while samples with less than 30 observations are always described as small samples. To summarize, while the Standard Deviation of a small sample (S) is used as the basis for estimating the Standard Deviation of the population (σ) utilizing equation (1.5), the Standard Deviation of a large sample is expressed directly as σ and the correction factor $[\sum(f)/(\sum(f)-1)]^{1/2}$ is not used.

1.2.5 The Coefficient of Variation

The Coefficient of Variation (v or V) is a further measure of the degree of variation or scatter within a sample. It is expressed as a percentage and provides a simple method of obtaining a measure of the correlation among a set of experimental results, such as concrete specimen that are tested to destruction to verify the strength of the structural concrete members in a building.

For a small sample: $v = [(S / \bar{x}) \times 100]\% \quad \ldots\ldots (1.6)$

For a large sample: $V = [(\sigma / \bar{x}) \times 100]\% \quad \ldots\ldots (1.7)$

Where: S = Standard Deviation of sample

σ = Standard Deviation of population

$\bar{x}$ = Mean of sample for (1.6) and population for (1.7)

Naturally, the smaller the value of v or V the better the correlation. At the same time, the appropriate interpretation of the Coefficient of Variation value is largely governed by the type of material being tested and the experimental procedure that was employed. In the case of concrete it is very difficult to achieve a v value below 10%, even under the most stringent experimental procedures.

1.2.6 What is a Standard Error?

It is accepted as a general rule in statistics, that *the scatter of means is always much less than the scatter of individual observations*. The reader may wish to verify this rule by comparing the individual and mean results of tossing a coin. If the Standard Deviation of individual observations in a population (σ) is known, then the best estimate of the standard error (or deviation) of the means of samples (σ_m) taken from the same population is given by:

$$\boldsymbol{\sigma_m = [\,\sigma / \sum(f)\,]^{1/2}} \quad \text{...} \quad \textbf{(1.8)}$$

Where: σ_m = Standard Error of Means

σ = Standard Deviation of single observations

$\sum(f)$ = total number of observations in sample

1.2.7 What are Confidence Limits?

So far we have used the parameters of small samples, such as the Mean ($\bar{x}$) and Standard Deviation (S), to predict the parameters of the populations from which these samples were obtained. Let us assume for a moment that the Standard Deviation (S) of a small sample of size 10 is 2.4. Then using equation 1.5, the Standard Deviation of the population is predicted to be:

$$\sigma = 2.4\,[\,10 / (10 - 1)\,]^{1/2} = \mathbf{2.53}$$

Of course, we have no reason to believe that the Standard Deviation of the population (σ) is exactly 2.53, we have simply estimated it to be very close to that value. It is frequently desirable to know the probability that a certain estimate based on a small sample is in fact correct. Similarly, we may wish to ascertain the actual probability that a certain observation (or mean) could be contained in a particular population. For example, if the lengths of a small sample of 10 tiles were measured to be:

7.30, 7.20, 7.25, 7.28, 7.32, 7.46, 7.50, 7.22, 7.54, and 7.28 IN

What is the probability that the Mean of the population (i.e., the entire stack of tiles from which the sample of 10 was selected at random) is 7.27 IN or less? We are able to calculate such probabilities by reference to the Normal Distribution curve (Figure 1.8), based on the following four criteria:

Criterion A: Any distribution (X_1) is said to have been standardized when it has been adjusted so that its Mean ($\bar{x}$) is zero and its Standard Deviation (σ) is 1.

Criterion B: A Normal Distribution with the Mean equal to zero and the Standard Deviation equal to unity is known as the Standard Normal Distribution.

Criterion C: The Standard Normal Variable (z) refers to the area under the Normal Distribution curve and is given by:

$$\pm z = (x_1 - \bar{x}) / \sigma \quad \text{.........} \quad (1.9)$$

Where: x_1 = a single observation

$\bar{x}$ = the Mean

σ = the Standard Deviation of the population

Criterion D: When considering the probable accuracy of the prediction of the Mean of the population, equation (1.9) is rewritten in the following form in terms of the calculated Mean ($\bar{x}$), the Mean (x') for which a probability is to be determined, and the Standard Deviation of Means (σ_m):

$$\pm z = (\bar{x} - x') / \sigma_m \quad \text{.........} \quad (1.10)$$

Where: $\bar{x}$ = the Mean

x' = a Mean

σ_m = the Standard Deviation of Means

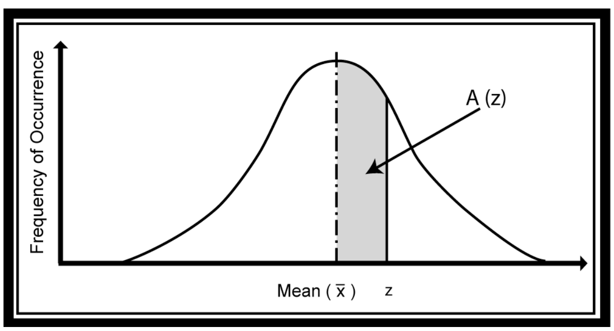

Figure 1.8: Standard Normal Distribution

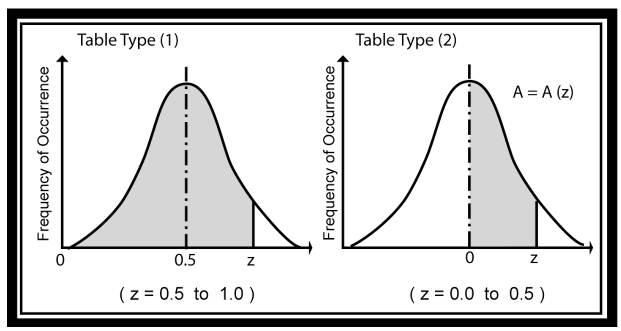

Figure 1.9: Normal Distribution table formats

Areas under the Normal Distribution curve are frequently needed and are therefore widely tabulated (Table 1.4).

Let us denote by *A(z)* the area under the Normal Distribution curve from *0* to *z*, where *z* is any number (i.e., *z* may have a positive, negative, or zero value). As shown in Figure 1.9, some Normal Distribution tables give the value of *A(z)* for positive values of *z* in steps of *0.01* from $z = 0$ to $z = 0.5$ and others give the value of *A(z)* from $z = 0.5$ to $z = 1.0$ depending on whether the area under the curve is measured from the Mean or from the left side. It is readily seen that Table 1.4 is of table type (1) in Figure 1.9 and therefore starts with an *A(z)* value of *0.5* (as opposed to *0.0*) in the top left hand corner of the table.

Summarizing, we conclude that equation (1.9) is always used to find the probability that a single random observation may occur in a population. Equation (1.10) is used to find the bounds of the Mean of a population.

Table 1.4: The Normal Probability Integral A(z)

The Normal Probability Integral A(z)										
z	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

1.2.8 Predicting the Strength of Concrete

On a large concrete dam construction project 121 concrete test cylinders were taken and subjected to compressive strength tests, with the results shown below. Find the probability that a random test cylinder will have a compressive strength of more than 1,800 psi?

Mean compressive strength ($\bar{x}$) = 2,160 psi
Standard Deviation (σ) = 252 psi
Random observation (x_1) = 1,800 psi

Apply equation (1.9): $\pm \mathbf{z} = (\mathbf{x_1} - \bar{\mathbf{x}}) / \boldsymbol{\sigma}$

$$\pm z = (1800 - 2160) / 252$$
$$\pm z = (-360) / 252$$
$$z = 1.43$$

From Table 1.4 we obtain a probability of *0.9236* for a *z* value of *1.43*. Thus the probability of obtaining a strength greater than 1,800 psi is 92.36% (i.e., approximately 92%).

For the same 121 concrete test cylinders of the above example, find the 95% confidence limits of the Mean compressive strength of all of the poured concrete deduced from this large sample of 121 test cylinders.

Sample Mean (x') = 2,160 psi
Standard Deviation (σ) = 252 psi

Step (1): Apply equation (1.8) to determine the Standard Deviation of the Mean (σ_m).

$$\sigma_m = [\sigma / \sum(f)]^{1/2}$$
$$\sigma_m = [252 / 121]^{1/2}$$
$$\boldsymbol{\sigma_m} = \mathbf{23\ psi}$$

Step (2): Apply equation (1.10) to determine the 95% confidence limits of the Mean compressive strength of the whole population ($\bar{x}$).

$$\pm z = (\bar{x} - x') / \sigma_m$$

Transposing equation (1.10) to make (x) the subject of the equation, we obtain:

$$\bar{x} = x' \pm z(\sigma_m)$$

The value of *z* is obtained for the required 95% probability as shown in Figure 1.10 to be *0.975*.

The corresponding *z* value for a probability of 0.975 is given in Table 1.4 as *1.96*, therefore substituting in the transposed equation:

$$\bar{x} = x' \pm z(\sigma_m)$$
$$\bar{x} = 2160 \pm (1.96 \times 23)$$
$$\bar{x} = 2160 \pm 45$$

Accordingly, the 95% confidence limits of the Mean compressive strength of all of the poured concrete in the dam are between **2,115 psi and 2,205** psi.

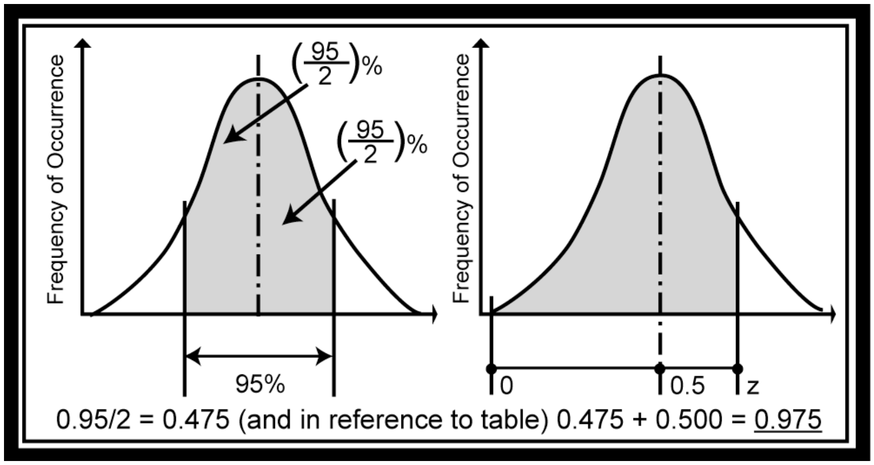

Figure 1.10: Confidence limits of the Mean compressive strength of concrete

1.3 Foundational Concepts in Physics

The chapters that follow assume some knowledge of the scientific concepts and principles that underlie current understanding of the nature of the physical phenomena that we refer to as heat, light and sound, and how these environmental stimuli are perceived by us human beings. Most of these stimuli have been studied for centuries as the various specialized fields of science emerged. In the following sections a few selected members of this foundational group of scientific principles are briefly explained in layperson terms. We will start with units of measurement because they are fundamental to all scientific and technical endeavors (Cardarelli 1997).

1.3.1 Units of Measurement

Measurement of length and volume became an important early concern as civilization evolved with an increasing focus on agriculture, trade, specialization, and collective aspirations that led to more and more community endeavors. The earliest length measurement of major consequence was most likely the Egyptian *cubit*, which was based on the length of the human arm from the elbow to the finger tips. While this provided a basis for measuring relatively short lengths such as those associated with plants, animals, and manmade artifacts, two additional needs soon surfaced. First, as communities grew in size and influence the need for standardization became paramount. By 2500 BC the Egyptians had already seen the need for the establishment of a Master Cubit made of marble. Second, as the roots of science emerged so also did the need for the accurate measurement of a host of additional quantities beyond length, area, volume, and time (Klein 1975).

The national standardization of units of measurement progressed somewhat more slowly than might have been expected. In England units of measurement were not effectively standardized until the early 13th Century. However, deviations and exceptions continued long thereafter. For example, total agreement on the volume measure, *gallon*, was not reached until the early 19th Century. Until then there existed the different ale, wine, and corn *gallons* (Connor 1987).

The US adopted the English system of weights and measures, with some specific exceptions. For example, the wine-gallon of 231 cubic inches was adopted in preference to the English *gallon* of 277 cubic inches. France officially adopted the metric system in 1799 with the *metre*[1] as the unit of length. The *metre* was defined as one ten-millionth part of the quarter of the circumference of the earth. Later this basis for defining a standard *metre* was replaced by a more exact and observable physical phenomenon[2], as the metric system became the *Systeme International (SI)* (i.e., Le Systeme International d'Unites) in 1960. Today, almost all nations with the notable exception of the US have adopted the SI system of units (United Nations 1955). However, even in the US the scientific community has to all intents and purposes unofficially migrated to the SI standard.

SI units of measurement: The SI standard is based on seven categories of units from which many other units are derived.

Category	Name	Abbreviation
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Each of these base units is clearly defined as a fraction of some measurable physical phenomenon. For example, the *kilogram* is based on the weight of a platinum-iridium cylinder maintained under constant environmental conditions in Sevres, France and the *second* is defined as the length of time taken by 9192631770 periods of vibration of the caesium-133 atom. SI units that are derived from these base unit categories include: the *farad (F)* for electrical capacitance; the *hertz (Hz)* for the frequency of a periodic vibration such as sound; the *joule (J)* for work and energy; the *newton (N)* for force; the *ohm (Ω)* for electric resistance; the *pascal (Pa)* for pressure; the *volt (V)* for electric potential; and the *watt (W)* the rate of doing work or power.

In addition, the SI standard utilizes specific prefixes to serve as convenient multiplication factors for increasing or reducing the relative size of these units in multiples of 1000. Commonly used prefixes are *K* for thousand, *M* for million, *G* for billion, *m* for thousandth, *µ* for millionth, and *n* for billionth. Therefore, *kW* stands for kilowatt or 1000 watt, and *mm* stands for millimeter or 1000th of a metre.

US system of units: Following the official adoption of the SI metric system of measurement by Britain in 1995, the US stands virtually alone with its continued use of what was originally known as the United Kingdom (UK) System of Measurements. With only a few specific differences the US system of measurements is the same as the pre-1995 UK system. However, whereas in the UK system the base measures of *yard,*

1 The US has adopted an alternative spelling of *meter*.

2 In the SI standard the *metre* is defined as the distance light travels in 1/299792458th of a second.

pound, and *gallon* were originally defined by physical standards, in the US system these are all now defined by reference to the SI *metre, kilogram,* and *litre*. The US system recognizes nine distinct categories of units, as follows[3]:

Length: *inch (IN), foot (FT), yard, furlong,* and *mile*.

Area: *square inch (SI), square foot (SF), acre, square mile* or *section, and township* (i.e., 36 square miles or 36 sections).

Volume: *cubic inch (CI), cubic foot (CF),* and *cubic yard*.

Capacity (dry): *pint, quart, peck,* and *bushel*.

Capacity (liquid): *fluid ounce, gill, pint, quart,* and *gallon*.

Mass: *grain, ounce, pound (LB), stone, hundredweight,* and *ton*.

Troy Weights: *grain, ounce, pennyweight,* and *pound (LB)*.

Apothecaries' Measures: *minim, dram, fluid ounce,* and *pint*.

Apothecaries' Weights: *grain, scruple, dram, ounce,* and *pound (LB)*.

It should be noted that among the mass units there are various versions of the *ton* unit and all of these are different from the original UK *ton*. A standard US *ton* is equal to 2000 LB instead of the original 2240 LB, while a US *ton* or *short ton* is equal to 2000 LB, a US *metric ton* is equal to 1000 LB, and a US *long ton* is equal to 2240 LB. To make matters even more confusing a US *measurement ton* has nothing to do with mass, but refers to a volume of 70 CF.

Conversion factors that must be applied to convert one unit of measurement to another are readily available in the form of published tables. Horvath (1986) has included conversion factors covering both historical and current units. Cardarelli (1997) provides more than 10,000 conversion factors in his more recent publication, which claims to be the most complete set of tables dealing with unit conversion.

1.3.2 Temperature Scales and Thermometers

Temperature provides a measure of the degree of hotness or coolness as perceived by our human senses. The desire to measure the precise degree of this sensation has led to a rich history of temperature scales and measurement instruments, commonly referred to as thermometers.

As might be expected the various physical states of water has served as a convenient set of reference points to the present day for defining alternative temperature scales. One of the earliest records of a temperature scale dates back to 170 AD when Galen, in his medical writings, proposed four degrees of heat and four degrees of cold on either side of boiling water and ice. In 1610 Galileo constructed a simple apparatus consisting of a glass tube with a bulb at one end and open at the other end. Holding the tube upright, he placed the open end into a container of wine, and extracted a small amount of the trapped air so that the wine would rise some distance above the level of the wine container inside the glass tube. The contraction and expansion of the air above the column of wine with changes in temperature would force the level of the wine in the glass tube to likewise rise and fall, correspondingly. The first attempt to

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[3] Where abbreviations are shown they refer to the abbreviations used throughout this book (in the absence of a standard US notational convention).

use a liquid, rather than a gas, for recording temperature is credited to Ferdinand II, Grand Duke of Tuscany in 1641. He proposed a device that held a quantity of alcohol in a sealed glass container with gradations marked on its stem. However, his device failed to reference the scale to a fixed point such as the freezing point of water.

In 1724, the Dutch instrument maker Gabriel Fahrenheit, used mercury as a temperature measuring medium. Mercury has several advantages as a thermometric medium. First, it has a relatively large coefficient of thermal expansion that remains fairly constant (i.e., linear) over a wide range of temperatures. Second, it retains its liquid form at temperatures well below the freezing point of water and well above the boiling point of water. Third, it does not easily separate into bubbles that might adhere to the glass surface as the column of mercury rises and falls with increasing and decreasing temperatures. On his scale, Fahrenheit fixed the boiling and freezing points of water to be 212° and 32°, respectively, providing an even 180 divisions in between. The Fahrenheit temperature scale was adopted as the basis of measuring temperature in the British system of units, which has now become the US system of units. Measurements recorded with this scale are referred to as *degrees Fahrenheit (°F)*.

In 1745, Carolus Linnaeus of Sweden proposed a temperature scale in which the freezing point of water is fixed as 0° and the boiling point of water is fixed at 100°, with 100 divisions in between these two reference points. This reversed the temperature scale that had been proposed a few years earlier by Anders Celsius, who had set 0° as the boiling point and 100° as the freezing point of water. Maintaining Linnaeus' reversal, the name *Centigrade* was replaced by *Celsius* in 1948[4]. The conversion of Fahrenheit to Celsius degrees and vice versa proceeds as follows:

$$°C = 5 / 9 \, (°F - 32) \quad \ldots\ldots \quad (1.11)$$

$$°F = 9 / 5 \, (°C) + 32 \quad \ldots\ldots \quad (1.12)$$

There is one other scale that has relevance to temperature. It is related to the concept of a *thermodynamic temperature*. In 1780, the French physician Jacques Charles (1746-1823) demonstrated that for the same increase in temperature all gases exhibit the same increase in volume. In other words, the coefficient of thermal expansion of all gases is very nearly the same. Subject to this fact it is possible to devise a temperature scale that is based on only a single fixed point, rather than the two fixed points that are necessary for the Fahrenheit and Celsius scales. This temperature is referred to as the *thermodynamic temperature* and is now universally accepted as the fundamental measure of temperature. The single fixed point in this temperature scale is an *ideal gas* pressure of zero, which is also defined as *zero temperature*. The unit of measurement on this scale is called *Kelvin*, named after Lord Kelvin (i.e., William Thompson 1824-1907). The symbol used is *K* without the degree (°) symbol. To convert degrees Celsius to Kelvin units we simply add 273.

$$K = °C + 273 \quad \ldots\ldots \quad (1.13)$$

$$K = 5 / 9 \, (°F - 32) + 273 \quad \ldots\ldots \quad (1.14)$$

$$°C = K - 273 \quad \ldots\ldots \quad (1.15)$$

[4] Apart from the name change there are also some very small differences in the precise temperature values of the freezing and boiling points of water. Under standard atmospheric conditions the boiling point of water is defined on the Celsius scale as 99.975°C, as opposed to 100° on the Centigrade scale.

$$°F = 9/5(K) - 241 \qquad \ldots\ldots \qquad (1.16)$$

1.3.3 Objective and Subjective Measurements

With very few exceptions, buildings are designed and constructed to be occupied by human beings. Therefore, in the study of building science we are concerned as much with how the human occupants of buildings perceive their environment as we are with the physical nature of the environment itself. While the perception of heat, light and sound is of course directly related to the stimuli that are received and processed in the human cognitive system, the measurement of what was received and what is perceived may differ widely. For example, while the amount of sound produced by a person speaking on a cell phone in a public place such as a restaurant can be measured objectively with a sound level meter, the degree of annoyance that this telephone conversation may cause to nearby customers depends very much on the sensitivity, current activity and emotional state of each person who is forced to overhear the telephone conversation. These individual perceptions are subjective reactions.

Objective information can normally be measured with an instrument or counted. It is typically information that is observable and factual. Examples include the measurement of light with a light meter, sound with a sound level meter and temperature with a thermometer. If the instrument is true and properly calibrated then the measurement of exactly the same sound should not vary from one sound level meter to another. However, the subjective perception of that sound may very widely from one person to another and even for the same person under different circumstances.

Both objective and subjective data can be collected in experiments or in assessing some particular aspect of a building environment. For example, based on complaints received about the stuffiness of a particular building space from the occupants of a new building, it may become necessary to conduct a survey of opinions to determine the degree of dissatisfaction. The data collected will likely be based on responses to a questionnaire and therefore subjective in nature. However, the survey may be followed by a systematic assessment of actual environmental conditions including measurement of the density of occupation, temperature, relative humidity, and degree of air movement in the space. All of these are objective measurements.

Methods that are normally used to collect objective data include: measurements taken with an instrument; recorded data (e.g., sound, light, heat, humidity, and air movement); and, direct objective measurements of a physical product, or natural event, phenomenon or object. Such measurements are reproducible and factual. On the other hand, the methods used for collecting subjective data are quite different in nature. They include ranking and rating methods, questionnaires, and interviews. The interpretation of subjective data must be undertaken with a great deal of caution because they are subject to human bias (Cushman and Rosenberg 1991). For this reason alone, it is considered good practice to collect both objective and subjective data during experiments, surveys, and assessments of environmental conditions involving human subjects.

1.3.4 Stress and Strain

The relationship between stress and strain is one of the fundamental concepts in the fields of material science and structural engineering. When a material is subjected to some kind of external force then it will in some manner respond by changing its state. For example, if we walk on a suspended platform, such as the concrete floor of a multistory building then the force applied by our weight will produce a physical strain within the concrete material. The resulting strain may result in a visible deflection of the floor. Similarly, if we blow air into a rubber balloon then the air pressure will result in a stretching of the balloon material, with the result that the balloon increases in size. If we continue to blow air into the balloon then eventually the strain in the rubber material will exceed the strength limit of the material and the balloon will burst.

In 1678 the English scientist Robert Hooke showed by means of a series of experiments that within the elastic range of a material the degree of strain produced is directly proportional to the amount of stress imposed. This relationship has come to be referred to as Hooke's Law and is easily verified with a very simple apparatus. If we freely suspend a thin metal wire from a nail at some height above the ground and then progressively load the lower end of the wire with more and more weights, we will observe that the wire will slightly increase in length after each weight has been added. When the increases in weight and length are plotted on graph paper a straight line will be the result, indicating a linear relationship between the load (i.e., stress) imposed and the resulting deflection (i.e., strain).

The relationship between stress (i.e., stimulus) and strain (i.e., reaction) applies generally to all kinds of situations, although the relationship is not necessarily linear when we move out of the field of material science into the biological domain. For example, in the field of building lighting increases in illumination level tend to increase the ability to see details. For this reason, the recommended task illumination level for fine machine work (e.g., sewing) is much higher than for casual reading. However, if we continue to increase the illumination level then a point will eventually be reached when the light level is so intense that we can no longer see any details. This condition is referred to as *disability glare*.

Similarly, the thermal environment can produce stresses that will produce physiological strain in the occupants of buildings. Examples of such strain include an increased heart rate, the dilation or constriction of blood vessels, and perspiration or shivering. It is important to note that the stress imposed by a building environment is cumulative. For example, slightly inadequate lighting conditions in a building space may not by themselves produce any observable human discomfort. However, when this condition is coupled with a slightly elevated room temperature and perhaps also a moderately excessive level of background noise, the resultant cumulative strain may exceed the comfort level of the building occupant by a disproportionally high degree.

1.3.5 Black Body Radiation

A *black body* is a concept in physics that the non-technical reader is unlikely to be familiar with. Since it is of significance in some aspects of the thermal environment, in particular in respect to the utilization of solar energy, and also in respect to artificial light sources, it warrants some explanation. When the temperature of any material is raised above the temperature of its surroundings it will radiate heat to its surroundings[5]. On the other hand, when an object is at a lower temperature than its surroundings then the surroundings will radiate heat to the object, which will absorb some of the heat and reflect and transmit the remaining heat. A *black body* is a theoretical object that absorbs all of the radiant energy that falls on it. It is an idealized concept because no such material exists in the real world. The material that comes closest is the graphite form of carbon, which absorbs close to 97% of the radiation that is incident on its surface.

Therefore, an ideal solar collector surface would be a *black body*. Water at any depth is for practical purposes considered to act as a *black body* and this phenomenon underlies the

[5] This of course applies only to those surfaces of the material that are not in direct contact with the surfaces of other objects. In the case of surfaces that touch each other heat is transferred from the object that is at a higher temperature to the object that is at a lower temperature by *conduction*.

principle of solar ponds. A solar pond is essentially a shallow pool of water that may or may not have a single glass sheet placed at a small distance above the water surface for added efficiency. Because of the near *black body* characteristics of the water, even without the single glazing, the pond acts as a very efficient absorber of solar heat radiation.

In the case of a standard flat-plate solar collector the upper surface of the metal plate is typically provided with a matte black finish. However, sometimes a more expensive *selective surface* coating with special heat absorbing characteristics is applied. A *selective surface* is a surface finish that has a large solar heat absorptance and a small solar emittance[6].

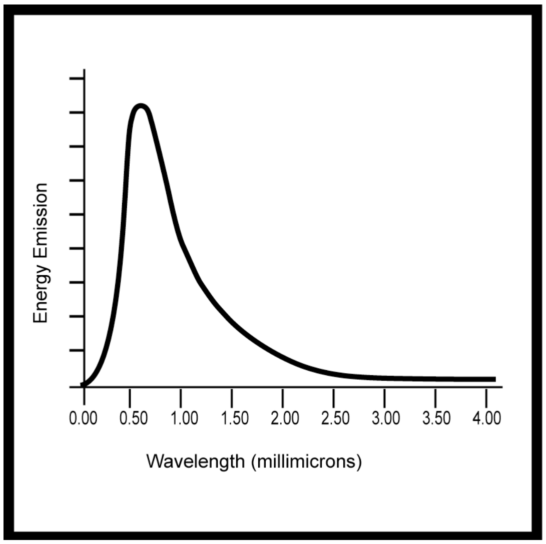

Figure 1.11: *Black Body* curve for 5000 K

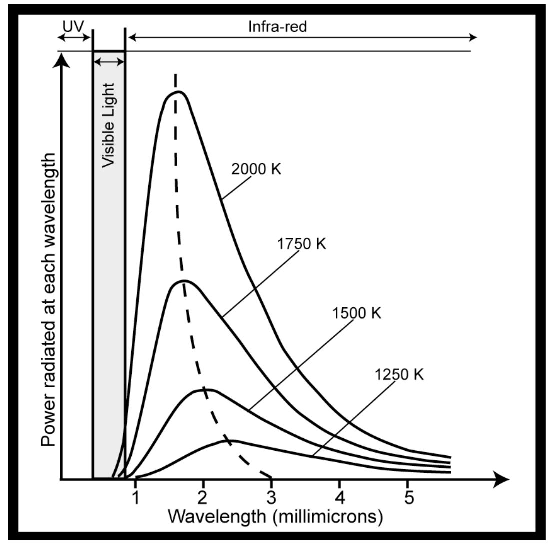

Figure 1.12: The *black body* radiation spectrum at increasing temperatures with the maximum emission wavelength moving toward the visible range

A *black body* is also a radiator of heat if its temperature is higher than the temperature of its surroundings. The radiation produced consists of a continuous spectrum of wavelengths. In fact, a *black body* at any temperature will emit energy at all wavelengths but to varying amounts. As shown in Figure 1.11, the theoretical energy emission curve is asymptotic[7] to the *X* axis. In other words, at very long wavelengths the curve never quite reaches zero emission. However, the actual amount of radiation varies both with the wavelength and the temperature of the *black body*. The precise radiation spectrum produced by a *black body* can be calculated mathematically and is referred to as the *black body radiation* for that temperature (Figure 1.12). Standard *black body radiation* curves have been drawn for each temperature based on the *Kelvin* temperature scale. In this way, for example, the spectral distribution of an artificial light source can be rated according to its equivalent *black body radiation* curve. This is referred to as

[6] The efficiency of the flat-plate solar collector increases if either the absorptance increases or the emittance decreases. A maximum efficiency is reached when the emittance is zero. In the case of a *selective surface* the absorptance is greater than the emittance, while for a *non-selective surface* the absorptance is equal to the emittance (Kreider and Kreith 1975, 42 and 96).

[7] An *asymptotic curve* never crosses the axis but comes infinitesimally close to the axis.

the *color temperature* of the light source, because it characterizes the color rendition properties[8] of the light source. As the temperature of the *black body* increases the maximum of the radiation spectrum moves toward the shorter wavelengths (Figure 1.12). Therefore, at some temperature the radiation spectrum of a *black body* will maximize the amount of light produced, because light is a narrow bandwidth within the full electromagnetic spectrum of radiation. This ideal color temperature is around *6,000 K*[9].

[8] The term *color rendition* refers to the appearance of a colored surfaces under a light source with a particular spectral distribution. For example, a red surface under a blue light will appear almost black because most of the blue light is absorbed by the red surface. However, a red light will accentuate the redness of a red surface because much of the red light will be reflected.

[9] As explained in a later chapter on artificial lighting *color temperature* must not be confused with the operating temperature of an artificial light source.